Nonlinear Set–Theoretic Localization of Cellular Phones

Joachim Horn[a], Uwe D. Hanebeck[b], Konrad Riegel †, Kai Heesche[a], and Werner Hauptmann[a]

[a]Siemens AG, Corporate Technology, Information and Comm., 81730 Munich, Germany
[b]Univ. Karlsruhe, Inst. of Computer Design and Fault Tolerance, 76128 Karlsruhe, Germany

ABSTRACT

Within the existing GSM standard, several measurements are available that can be used for estimating the position of a cellular phone. First, the timing advance (TA) gives an estimate for the distance to the serving base station. Second, the signal strengths (RXLEV) of neighbouring base stations can also be interpreted as distance information. Both TA and RXLEV are subject to measurement errors caused for example by shadowing, reflections, and fast fading. Thus, a nonlinear set–theoretic estimation technique based on pseudo ellipsoids is applied. The uncertainty regions in the original space defined by the measurements are transformed into a hyperspace of higher dimension and described by pseudo ellipsoids. An approximation of the set intersection of the pseudo ellipsoids can be calculated recursively by a linear set–theoretic filter. The resulting pseudo ellipsoid is transformed back into the original space, and the position estimate is calculated as center of gravity of the resulting uncertainty region. The algorithm is evaluated based on the data of an extensive field trial in a rural area. Compared to pure cell ID, the accuracy is significantly increased by using TA and RXLEV, reducing the mean error by half.

Keywords: Nonlinear Filtering, Set–Theoretic Filtering, Localization, Cellular Phones, GSM Networks

1. INTRODUCTION

With the increasing demand to provide location based services already in existing GSM networks,[5] there is a significant interest in mobile positioning approaches operating on the basis of installed network infrastructure and legacy cellular phones. Within the existing GSM standard, several measurements are available that can be used for estimating the position of a cellular phone. First, the timing advance (TA) gives an estimate for the distance to the serving base station. Second, the signal strengths (RXLEV) of neighbouring base stations can also be interpreted as distance information. Both TA and RXLEV are subject to measurement errors caused for example by shadowing, reflections, and fast fading. Thus, stochastic algorithms[9,10,13] as well as pattern recognition techniques[14] have been applied. Here, a set–theoretic filter approach is proposed.

Since the measurement equations are nonlinear and the uncertainties are large, a standard filtering approach based on linearization of the measurement equation cannot be applied. Instead, a new nonlinear set–theoretic estimation technique based on pseudo ellipsoids[8] is used.

Estimating the state of a dynamic system based on a sequence of uncertain measurements is a standard problem in many applications. Usually, this problem is approached in a stochastic setting. Alternatively, set–theoretic methods can be used by assuming a priori bounds on the uncertainties. Estimation then consists of constructing sets of possible states, which are consistent with the a priori bounds and the measurements.[1,2,19]

Most work has been done in set–theoretic state estimation for linear systems.[17,18,21,22] Typical applications are in the field of speech processing[3,4] and robotics,[7,16] where the differences between stochastic and set–theoretic estimation have been studied intensively.[6]

Further author information: (Send correspondence to Joachim Horn.)
Joachim Horn: E-mail: Joachim.P.Horn@siemens.com, Telephone: +49 (0)89 636 50927
Uwe D. Hanebeck: E-mail: Uwe.Hanebeck@ieee.org, Telephone: +49 (0)721 608 3909
Konrad Riegel died in an avalanche in January 2003 while skiing in the Alps.
Kai Heesche: E-mail: Kai.Heesche@siemens.com, Telephone: +49 (0)89 636 53578
Werner Hauptmann: E-mail: Werner.Hauptmann@siemens.com, Telephone: +49 (0)89 636 44427

In the case of nonlinear systems, the complex sets resulting from the estimation procedure are either approximated by simple–shaped sets, e.g. ellipsoids, boxes,[12,15] polytopes,[20] or by the union of simple sets.[11,12]

Set theoretic estimators similar in concept to the Extended Kalman Filter (EKF) have also been pursued.[20] As in the EKF, the nonlinear state equations are linearized about the current state estimate. Unlike the EKF, linearization errors are not neglected, but rather considered as additional exogeneous disturbances. Estimation is performed recursively and provides polytopes as approximation of the posterior feasible set.

Estimators not relying on linearization of the nonlinear state equation have also been proposed and work e.g. by recursively calculating the smallest axis–aligned box enclosing the posterior feasible set.[15] More complex approaches characterize the posterior feasible set by enclosing it between internal and external unions of boxes on the basis of interval analysis.[11,12]

In this paper, a new nonlinear filtering algorithm for nonlinear systems is applied, that does not rely in any way on linearization. In addition, the approach is not based on a grid or on propagating particles, but provides a finite–dimensional closed–form representation of the resulting complex–shaped sets. This includes nonconvex sets or sets that are not even connected. When applying this filter recursively to a sequential stream of measurements, the size of the analytical representation of the resulting sets does not grow with the number of measurements.

The key idea of the filter is to transform the original N–dimensional space S to an L–dimensional hyperspace S^* with $L > N$. This results in an N–dimensional manifold U^*, called the universal manifold, in the L–dimensional transformed space S^*. Complex–shaped subsets of the original N–dimensional space are then represented by N–dimensional submanifolds of U^* in the space S^*. These submanifolds are defined by the intersection of L–dimensional simple–shaped sets, e.g. ellipsoids, with the universal manifold U^*. Furthermore, the nonlinear measurement equation is transformed to a linear one in the hyperspace S^*. Hence, nonlinear filtering can be performed by a linear filter operating in the transformed space S^*.

In section 2, the nonlinear set–theoretic estimation technique is presented. In section 3, this technique is applied to the position estimation of cellular phones using TA and RXLEV measurements. Experimental results of an extended field trial in a rural area are presented in section 4.

2. RECURSIVE NONLINEAR SET–THEORETIC ESTIMATION

2.1. Problem Formulation

Consider a nonlinear discrete–time dynamic system with system state $\underline{x}_k$ (not directly observable) at time step k. Measurements $\hat{\underline{z}}_k$ of the system output are taken at time instants $k = 1, 2, \ldots$ according to the nonlinear measurement equation

$$\hat{\underline{z}}_k = \underline{h}_k(\underline{x}_k) + \underline{v}_k$$

with measurement uncertainty $\underline{v}_k$, which represents exogeneous noise sources or model parameter uncertainties.

The uncertainties $\underline{v}_k$, $k = 1, 2, \ldots$, are assumed to be bounded by a known set $\mathcal{V}_k$ according to $\underline{v}_k \in \mathcal{V}_k$. The set can be of complicated shape, i.e., can be nonconvex or not connected.

The goal is to estimate at each time instant k the state $\underline{x}_k$ based on *all* available measurements $\hat{\underline{z}}_l$ for $l = 1, 2, \ldots, k$. Of course, a recursive estimation procedure is preferred, which calculates a state estimate based on the estimate at the previous time step and the current measurement. Therefore, it is not required to store and reprocess all measurements. Furthermore, instead of trying to construct point estimates, we prefer to calculate at each time instant k all states that are compatible with the measurements and their corresponding uncertainties.

On a theoretical level, the problem can easily be solved: Let $\mathcal{X}^s_{k-1}$ denote the set of all states compatible with all the measurements up to time step $k-1$ and their respective uncertainties. Furthermore, $\mathcal{X}^m_k$ denotes the set of states defined solely by the measurement *at* time k according to

$$\mathcal{X}^m_k = \{\underline{x}_k : \hat{\underline{z}}_k - \underline{h}_k(\underline{x}_k) \in \mathcal{V}_k\} \quad .$$

Then the estimate $\mathcal{X}_k^s$ is given by the intersection

$$\mathcal{X}_k^s = \mathcal{X}_{k-1}^s \cap \mathcal{X}_k^m \quad .$$

However, representing these sets in practical applications at least approximately by a finite set of parameters is not a trivial task. On one hand, the parameter set should not be too large, even more, the approximation should degrade gracefully with a decreasing number of parameters. On the other hand, the number of parameters should not be permanently growing with an increasing number of incoming measurements. Hence, the remainder of this section is concerned with a new parametric representation of complex–shaped sets and an efficient procedure for calculating the corresponding parameters.

2.2. Pseudo Ellipsoids

The key idea is to represent an uncertainty $\mathcal{X}_k$ with a complicated shape in the N–dimensional original space S by a simpler shaped uncertainty $\mathcal{X}_k^*$ in an L–dimensional hyperspace S^* with $L > N$. Points $\underline{x}_k$ in S are related to points $\underline{x}_k^*$ in S^* via a nonlinear transformation $\underline{T}(.)$ according to

$$\underline{x}_k^* = \underline{T}(\underline{x}_k) = \Big[T_1(\underline{x}), T_2(\underline{x}), \dots, T_L(\underline{x})\Big]^T \quad .$$

Hence, $\underline{T}(.)$ defines an N–dimensional manifold U^* in an L–dimensional space.

In addition, L–dimensional sets $\mathcal{X}_k^*$ of simple shape are defined in the transformed space S^*. Here, ellipsoidal sets according to

$$\mathcal{X}_k^* = \left\{ \underline{x}_k^* : (\underline{x}_k^* - \hat{\underline{x}}_k^*)^T (\mathbf{X}_k^*)^{-1} (\underline{x}_k^* - \hat{\underline{x}}_k^*) \le 1 \right\}$$

are used, where $\hat{\underline{x}}_k^*$ is the ellipsoid midpoint and $\mathbf{X}_k^*$ is a symmetric positive definite matrix.

The intersection of an ellipsoid $\mathcal{X}_k^*$ with the manifold U^* defines a submanifold of U^*, which, in turn, defines a complicated set in the original space S.

REMARK 2.1 A complex–shaped set in the original space S is defined by *both* the transformation $\underline{T}(.)$ *and* the pseudo ellipsoid $\mathcal{X}_k^*$.

In many cases, the type of transformation $\underline{T}(.)$ results directly from the nonlinearities considered. For example, when considering polynomial nonlinearities, a polynomial transformation is used. For trigonometric nonlinearities, a trigonometric transformation can be used.

2.3. Filtering

Based on the concept of pseudo ellipsoids, which represent complex–shaped sets in the original space S by pseudo ellipsoids in the hyperspace S^*, the nonlinear filter step can now be performed by a linear filter in the hyperspace S^*. For that purpose, a pseudo–linear expansion of the nonlinear measurement equation $\underline{h}_k(.)$ is performed according to

$$\underline{h}_k(\underline{x}_k) = \mathbf{H}_k^* \underline{x}_k^* + \underline{e}_k^h \approx \mathbf{H}_k^* \underline{x}_k^* \quad ,$$

where $\underline{e}_k^h$ represents the approximation error defined by $\underline{e}_k^h = \underline{h}_k(\underline{x}_k) - \mathbf{H}_k^* \underline{x}_k^*$.

In general, the expansion can be enhanced by an additional nonlinear transformation $\underline{g}(.)$ of the measurements according to

$$\underline{g}(\hat{\underline{z}}_k - \underline{v}_k) = \underline{g}(\underline{h}_k(\underline{x}_k)) \quad .$$

The left hand side can be approximated by

$$\underline{g}(\hat{\underline{z}}_k - \underline{v}_k) = \hat{\underline{z}}_k^* - \mathbf{G}_k^* \underline{v}_k^* - \underline{e}_k^{v,*} \approx \hat{\underline{z}}_k^* - \mathbf{G}_k^* \underline{v}_k^* \quad ,$$

where $\hat{\underline{z}}_k^*$ and $\mathbf{G}_k^*$ are nonlinear functions of $\hat{\underline{z}}_k$ and $\underline{v}_k^*$ is a nonlinear function of $\underline{v}_k$. $\underline{e}_k^{v,*}$ accounts for the approximation error.

The right hand side is again approximated according to

$$\underline{g}(\underline{h}_k(\underline{x}_k)) = \mathbf{H}_k^* \underline{x}_k^* + \underline{e}_k^{h,*} \approx \mathbf{H}_k^* \underline{x}_k^*$$

with approximation error $\underline{e}_k^{h,*}$. As a result, the measurement equation in the hyperspace is obtained according to

$$\underline{z}_k^* = \mathbf{H}_k^* \underline{x}_k^* + \underbrace{\underline{e}_k^{h,*} + \mathbf{G}_k^* \underline{v}_k^* + \underline{e}_k^{v,*}}_{\underline{w}_k^*}$$

with $\underline{w}_k^*$ representing the total uncertainty.

Let the set of all predicted states be given by the set $\mathcal{X}_k^p$, which is defined in the transformed space S^* by

$$\mathcal{X}_k^{p,*} = \left\{ \underline{x}_k^* : (\underline{x}_k^* - \hat{\underline{x}}_k^{p,*})^T (\mathbf{E}_k^{p,*})^{-1} (\underline{x}_k^* - \hat{\underline{x}}_k^{p,*}) \leq 1 \right\} \quad .$$

Furthermore, let $\underline{w}_k^*$ be bounded by the set

$$\mathcal{W}_k^* = \left\{ \underline{w}_k^* : (\underline{w}_k^*)^T (\mathbf{W}_k^*)^{-1} \underline{w}_k^* \leq 1 \right\} \quad .$$

Then, the set defined by the measurement is given by

$$\mathcal{X}_k^{m,*} = \left\{ \underline{x}_k^* : (\hat{\underline{z}}_k^* - \mathbf{H}_k^* \underline{x}_k^*)^T (\mathbf{W}_k^*)^{-1} (\hat{\underline{z}}_k^* - \mathbf{H}_k^* \underline{x}_k^*) \leq 1 \right\} \quad .$$

The fusion result is given by a set $\mathcal{X}_k^{s,*}$ (again an ellipsoid in the transformed space, but a set of complicated shape in the original space!) that contains the intersection of the ellipsoids $\mathcal{X}_k^{p,*}$ and $\mathcal{X}_k^{m,*}$. Hence, $\mathcal{X}_k^{s,*}$ is obtained by a linear set–theoretic filter in the hyperspace S^*[19]

$$\mathcal{X}_k^{s,*} = \left\{ \underline{x}_k^* : (\underline{x}_k^* - \hat{\underline{x}}_k^{s,*})^T (\mathbf{E}_k^{s,*})^{-1} (\underline{x}_k^* - \hat{\underline{x}}_k^{s,*}) \leq 1 \right\}$$

with

$$\hat{\underline{x}}_k^{s,*} = \hat{\underline{x}}_k^{p,*} + \lambda_k^* \mathbf{E}_k^{p,*} (\mathbf{H}_k^*)^T \left\{ \mathbf{W}_k^* + \lambda_k^* \, \mathbf{H}_k^* \mathbf{E}_k^{p,*} (\mathbf{H}_k^*)^T \right\}^{-1} \left(\hat{\underline{z}}_k^* - \mathbf{H}_k^* \hat{\underline{x}}_k^{p,*} \right)$$

and

$$\begin{aligned} \mathbf{E}_k^{s,*} &= d_k^* \, \mathbf{P}_k^{s,*} \quad , \\ \mathbf{P}_k^{s,*} &= \mathbf{E}_k^{p,*} - \lambda_k^* \, \mathbf{E}_k^{p,*} (\mathbf{H}_k^*)^T \left\{ \mathbf{W}_k^* + \lambda_k^* \, \mathbf{H}_k^* \mathbf{E}_k^{p,*} (\mathbf{H}_k^*)^T \right\}^{-1} \mathbf{H}_k^* \mathbf{E}_k^{p,*} \quad , \end{aligned}$$

where

$$d_k^* = 1 + \lambda_k^* - \lambda_k^* \left(\hat{\underline{z}}_k^* - \mathbf{H}_k^* \hat{\underline{x}}_k^{p,*} \right)^T \left\{ \mathbf{W}_k^* + \lambda_k^* \, \mathbf{H}_k^* \mathbf{E}_k^{p,*} (\mathbf{H}_k^*)^T \right\}^{-1} \left(\hat{\underline{z}}_k^* - \mathbf{H}_k^* \hat{\underline{x}}_k^{p,*} \right) \quad .$$

Using this form of bounding ellipsoid for the exact intersection of $\mathcal{X}_k^{p,*}$, $\mathcal{X}_k^{m,*}$ in the transformed space S^* offers the advantage that the resulting set $\mathcal{X}_k^s(\lambda_k)$ *in the original space* S possesses the following property, which is desirable in applications in the sense, that no new uncertainty is introduced:

LEMMA 2.1 *If the two sets $\mathcal{X}_k^p$ und $\mathcal{X}_k^m$ overlap, the filtering result $\mathcal{X}_k^s(\lambda_k)$ contains the exact intersection $\mathcal{X}_k^p \cap \mathcal{X}_k^m$ and is itself contained in their union $\mathcal{X}_k^p \cup \mathcal{X}_k^m$. Hence, it holds*

$$(\mathcal{X}_k^p \cap \mathcal{X}_k^m) \subset \mathcal{X}_k^s(\lambda_k) \subset (\mathcal{X}_k^p \cup \mathcal{X}_k^m)$$

for all $\lambda_k \in [0, \infty]$.

The fusion parameter λ_k is selected in such a way, that a certain measure of the size of the set $\mathcal{X}_k^s$ is minimized.

3. NONLINEAR SET–THEORETIC LOCALIZATION OF CELLULAR PHONES

In GSM networks,[5] the mobile station sends a measurement report to the serving base station every 480 ms. Although these measurement reports are not intended for localization of the cellular phone, some of the data can be used for position estimation. The timing advance (TA) measures the time–of–flight from the serving base station to the mobile station. Using the line–of–sight assumption, the TA thus gives an estimate for the distance of the mobile station to the serving base station, with a quantization uncertainty of 554 m. The signal strength (RXLEV) of up to six neighbouring base stations can also be interpreted as distance of the mobile station to the respective base station. The base stations are identified using the cell ID, and the positions of the base stations are known.

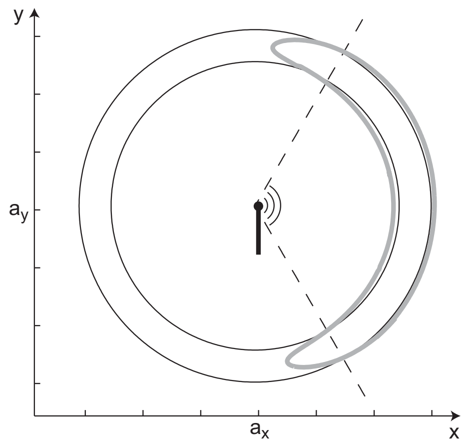

Figure 1. Approximation of the uncertainty region defined by the TA and the antenna characteristics.

The TA measurement defines a ring as uncertainty region. Using the orientation and characteristics of the serving antenna, this region can further be reduced, e.g. to a 120^o ring segment. Since the RXLEV measurements are subject to large errors caused by shadowing, reflections, and fast fading, they are interpreted as specification of a maximum distance to the respective antenna, thus defining a circle. The final region of uncertainty is then given by the intersection of the ring segment and the circles.

Since the exact intersection is of complex shape and hard to determine, the approximative nonlinear filtering technique presented in the previous section is used.

The uncertainty region defined by the TA measurement is given by the ring

$$R_i^2 \leq (x - a_x)^2 + (y - a_y)^2 \leq R_o^2 \ ,$$

where the inner radius R_i and the outer radius R_o are defined by the (quantized) TA value, with $R_o = R_i + 554\,\mathrm{m}$, and the center a_x, a_y corresponds to the position of the serving base station. This uncertainty region may be expressed as

$$\frac{R_o^2 + R_i^2}{2} = (x - a_x)^2 + (y - a_y)^2 + w$$

and is thus given as

$$\frac{R_o^2 + R_i^2}{2} = -2a_x x_1^* - 2a_y x_2^* + x_3^* + x_5^* + a_x^2 + a_y^2 + w$$

in the hyperspace

$$\underline{x}^* = \left[x,\ y,\ x^2,\ xy,\ y^2\right]^T = \left[x_1^*,\ x_2^*,\ x_3^*,\ x_4^*,\ x_5^*\right]^T \ ,$$

where the uncertainty $w^* = w$ is bounded by the interval

$$\mathcal{W}^* = \left[-\left(\frac{R_o^2 - R_i^2}{2} \right), \left(\frac{R_o^2 - R_i^2}{2} \right) \right] .$$

This defines the measurement equation

$$z^* = \mathbf{H}^* \underline{x}^* + w^* = \frac{R_o^2 + R_i^2}{2} - a_x^2 - a_y^2$$

with

$$\mathbf{H}^* = \left[-2a_x, \ -2a_y, \ 1, \ 0, \ 1 \right]$$

and the uncertainty set

$$\mathcal{X}^{m,*} = \left\{ \underline{x}^* : (\hat{z}^* - \mathbf{H}^* \underline{x}^*)^T \left(\left(\frac{R_o^2 - R_i^2}{2} \right)^2 \right)^{-1} (\hat{z}^* - \mathbf{H}^* \underline{x}^*) \leq 1 \right\}$$

in the hyperspace for the TA measurement.

The measurement equation and the uncertainty set for the antenna characteristics and the RXLEV measurements are defined in a similar way. Fig. 1 shows the approximation of the ring segment defined by the measured TA and the antenna characteristics. This uncertainty region in the original space corresponds to a pseudo ellipsoid in the hyperspace. Fig. 2 shows the corresponding uncertainty regions in the original space when the three RXLEV measurements to the neighbouring base stations 1 to 3 are recursively used to update the uncertainty region. Note that the uncertainty regions in the original space are only shown here to clarify the behaviour of the proposed algorithm, they are not used in the calculation. Finally, the position estimate is calculated as center of gravity of the resulting uncertainty region. Here, grid points in the original space are transformed into the hyperspace and tested if they are elements of the resulting uncertainty ellipsoid. This numerical calculation of the center of gravity avoids transforming the uncertainty ellipsoid from the hyperspace back into the original space.

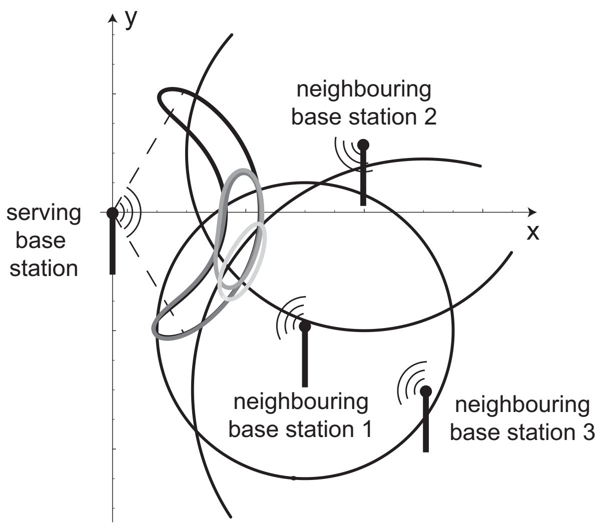

Figure 2. Approximation of the set intersection when three RXLEV measurements are recursively used to update the uncertainty region.

4. EXPERIMENTAL RESULTS

A data set from a field trial in a rural area was used for evaluating the algorithm. The data set consists of about one hundred thousand measurement reports, measured in a rural area of about 10 km × 8 km. For each measurement report, the reference position was measured by GPS. Fig. 3 shows the GPS positions as well as the positions of the three *serving* base stations.

The maximum radius as a function of signal strength was derived from maps of predicted signal strength. For each measurement report, an estimate of the position of the cellular phone was calculated using the proposed nonlinear set–theoretic algorithm. The algorithm was implemented in MATLAB, the average time for calculation of the estimate on a standard PC was about 10 ms. The mean error in position of the proposed algorithm was 774 m, while the mean error of pure cell ID, i.e., using the position of the serving base station as estimate, was 1483 m. Thus, the accuracy of position estimation has been significantly improved by using TA and RXLEV.

The accuracy could further be improved by increasing the dimension of the hyperspace. Nevertheless, the actual choice of the dimension seems to be a good compromise regarding accuracy and computational effort.

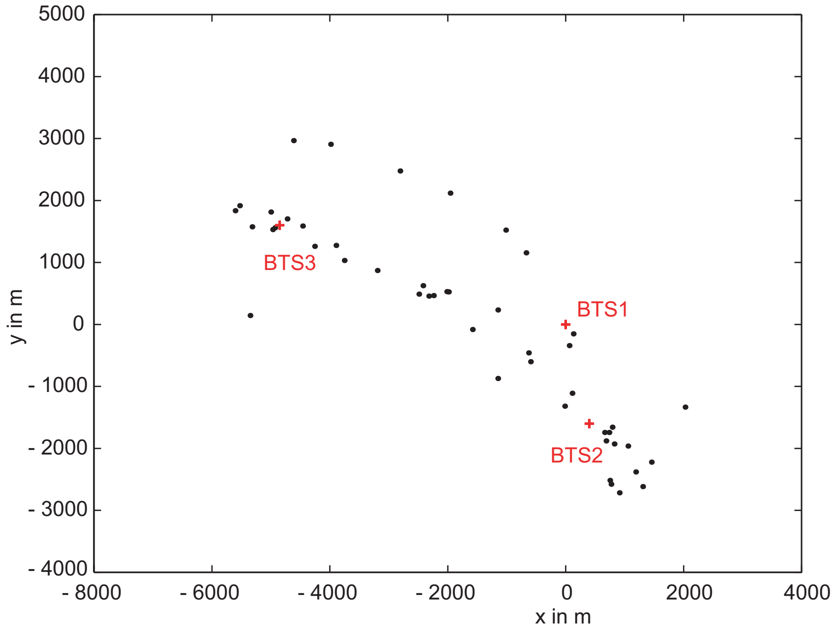

Figure 3. GPS positions of the measurement reports and positions of the serving base stations of the field trial.

5. CONCLUSIONS

A nonlinear set–theoretic approach for position estimation of cellular phones in a GSM network using cell ID, TA, and RXLEV has been proposed. The uncertainty regions in the original space defined by the measurements are transformed into a hyperspace of higher dimension and described by pseudo ellipsoids. Thus, an approximation of the set intersection of the pseudo ellipsoids can be calculated recursively by a linear set–theoretic filter. The resulting pseudo ellipsoid is transformed back into the original space, and the position estimate is calculated as center of gravity of the resulting uncertainty region. The algorithm was evaluated based on the data of an extensive field trial in a rural area. Compared to pure cell ID, the accuracy was significantly increased, reducing the mean error by half.

REFERENCES

1. F. L. Chernousko, *State Estimation for Dynamic Systems*, CRC Press, 1994.
2. P. L. Combettes, "The Foundations of Set Theoretic Estimation", *Proceedings of the IEEE*, Vol. 81, No. 2, pp. 182–208, 1993.
3. J. R. Deller, "Set Membership Identification in Digital Signal Processing", *IEEE ASSP Magazine*, Vol. 6, pp. 4–20, 1989.
4. J. R. Deller, M. Nayeri, S. F. Odeh, "Least–Squares Identification with Error Bounds for Real–Time Signal Processing and Control", *Proceedings of the IEEE*, Vol. 81, No. 6, pp. 815–849, 1993.
5. J. Eberspächer, H. J. Vögel, *GSM — Switching, Services and Protocols*, Wiley, 1999.
6. G. D. Hager, S. P. Engelson, S. Atiya, "On Comparing Statistical and Set–Based Methods in Sensor Data Fusion", *Proceedings of the 1993 IEEE International Conference on Robotics and Automation*, Atlanta, Georgia, pp. 352–358, 1993.
7. U. D. Hanebeck, G. Schmidt, "Set–theoretic Localization of Fast Mobile Robots Using an Angle Measurement Technique", *Proceedings of the 1996 IEEE International Conference on Robotics and Automation (ICRA'96)*, Minneapolis, Minnesota, pp. 1387–1394, 1996.
8. U. D. Hanebeck, "Recursive Nonlinear Set–Theoretic Estimation Based on Pseudo–Ellipsoids", *Proceedings of the IEEE Conference on Multisensor Fusion and Integration for Intelligent Systems*, Baden–Baden, Germany, pp. 159–164, 2001.
9. M. Hellebrandt, R. Mathar, M. Scheibenbogen, "Estimating Position and Velocity of Mobiles in a Cellular Radio Network", *IEEE Transactions on Vehicular Technology*, Vol. 46, No. 1, pp. 65–71, 1997.
10. M. Hellebrandt, R. Mathar, "Location Tracking of Mobiles in Cellular Radio Networks", *IEEE Transactions on Vehicular Technology*, Vol. 48, No. 5, pp. 1558–1562, 1999.
11. L. Jaulin and E. Walter, "Set Inversion via Interval Analysis for Nonlinear Bounded–Error Estimation", *Automatica*, Vol. 29, No. 4, pp. 1053–1064, 1993.
12. L. Jaulin, "Interval Constraint Propagation with Application to Bounded–Error Estimation", *Automatica*, Vol. 36, No. 10, pp. 1547–1552, 2000.
13. O. Kennemann, "Continuous Location of Moving GSM Mobile Stations by Pattern Recognition Techniques", *Proceedings of the Fifth IEEE International Symposium on Personal, Indoor and Mobile Radio Communications (PIMRC'94)*, Den Haag, Netherlands, pp. 630–634, 1994.
14. S. Mangold, S. Kyriazakos, "Applying Pattern Recognition Techniques Based on Hidden Markov Models for Vehicular Position Location in Cellular Networks". *Proceedings of the IEEE 50th Vehicular Technology Conference*, Amsterdam, Netherlands, pp. 780–784, 1999.
15. M. Milanese, A. Vicino, "Estimation Theory for Nonlinear Models and Set Membership Uncertainty", *Automatica*, Vol. 27, No. 2, pp. 403–408, 1991.
16. A. Preciado, D. Meizel, A. Segovia, M. Rombaut, "Fusion of Multi–Sensor Data: A Geometric Approach", *Proceedings of the 1991 IEEE International Conference on Robotics and Automation*, Sacramento, California, pp. 2806–2811, 1991.
17. A. K. Rao, Y.–F. Huang, S. Dasgupta, "ARMA Parameter Estimation Using a Novel Recursive Estimation Algorithm with Selective Updating", *IEEE Transactions on Signal Processing*, Vol. 38, No. 3, pp. 447–457, 1990.
18. A. Sabater, F. Thomas, "Set Membership Approach to the Propagation of Uncertain Geometric Information", *Proceedings of the 1991 IEEE International Conference on Robotics and Automation*, Sacramento, California, pp. 2718–2723, 1991.
19. F. C. Schweppe, *Uncertain Dynamic Systems*, Prentice–Hall, 1973.
20. J. S. Shamma, K.–Y. Tu, "Approximate Set–Valued Observers for Nonlinear Systems", *IEEE Transactions on Automatic Control*, Vol. 42, No. 5, pp. 648–658, 1997.
21. A. Vicino, G. Zappa, "Sequential Approximation of Feasible Parameter Sets for Identification with Set Membership Uncertainty", *IEEE Transactions on Automatic Control*, Vol. 41, No. 6, pp. 774–785, 1995.
22. E. Walter, H. Piet-Lahanier, "Exact Recursive Polyhedral Description of the Feasible Parameter Set for Bounded Error Models", *IEEE Transactions on Automatic Control*, Vol. 34, No. 8, pp. 911–915, 1989.